Math 1552

Review of Week 2

Math 1552 lecture slides adapted from the course materials
By Klara Grodzinsky (GA Tech, *School of Mathematics*, Summer 2021)

Review Question: Which integrals can we evaluate *by parts*?

$$\text{(A)} \int \frac{x^2}{1+x^3}\,dx$$

$$\text{(B)} \int \frac{1}{x} e^{\ln x}\,dx$$

$$\text{(C)} \int x^5 e^{x^3}\,dx$$

$$\text{(D)} \int x \tan^{-1}(x)\,dx$$

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Section 8.3: Powers and Products of Trigonometric Functions

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Today’s Goal:

- Use trigonometric formulas to reduce more difficult integrals until we can perform a *u*-substitution.

- Idea: rewrite the function in terms of just one trig function after “breaking off” its derivative for a *u*-substitution

Useful Trig Identities

$$(*)\sin^2 x + \cos^2 x = 1$$

$$(*)1 + \tan^2 x = \sec^2 x$$

$$(*)\sin^2 x = \frac{1}{2}\left[1 - \cos(2x)\right]$$

$$(*)\cos^2 x = \frac{1}{2}\left[1 + \cos(2x)\right]$$

$$(*)\sin(2x) = 2\sin x \cos x$$

$$\sin x \cos y = \frac{1}{2}\left[\sin(x - y) + \sin(x + y)\right]$$

$$\sin x \sin y = \frac{1}{2}\left[\cos(x - y) - \cos(x + y)\right]$$

$$\cos x \cos y = \frac{1}{2}\left[\cos(x - y) + \cos(x + y)\right]$$

$$\tan^2 x + 1 = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

(Where do these come from?)

Special cases: x=at, y=bt

Example 1.1: Evaluate the following integral $\int \mathbf{tan^3(x)dx}$

Example 1.2: Evaluate the following integral: $\int \cos^2(x)\cot(x)dx$

Example 1.3:

Evaluate the following integral: $\int \sin^4(x)dx$

Example 2.1: Evaluate: $\int \tan^3(x) \sec^3(x) dx$

Example 2.2: Evaluate: $\int \sec^3(x)dx$

Evaluate the integral.

$$\int \sin^2(x)\cos^3(x)\,dx$$

$$(A)\ \frac{1}{5}\sin^5(x) + C$$

$$(B)\ \frac{1}{3}\sin^3(x) - \frac{1}{5}\sin^5(x) + C$$

$$(C)\ \frac{1}{12}\sin^3(x)\cos^4(x) + C$$

$$(D)\ -\frac{1}{3}\cos^3(x) + \frac{1}{5}\cos^5(x) + C$$

Extra Problem: Evaluate the integral. $\displaystyle\int \frac{\sec^4(4x)}{\tan^9(4x)}\,dx$

Extra problem: Evaluate the integral. $\int \sin(5x)\cos(3x)\,dx$

Hint: $\sin(5x)\cos(3x) = \frac{1}{2}\left(\sin(2x) + \sin(8x)\right)$

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Section 8.4: Trigonometric Substitution

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Today's Learning Goals

- Identify which types of integrals can be solved with a trigonometric substitution
- Learn which substitution matches which general form
- Evaluate integrals using the method of trigonometric substitution

Trigonometric Substitutions

We use a trig substitution when no other integration method will work, and when the integral contains one of these terms:

$$a^2 - x^2$$

$$x^2 - a^2$$

$$a^2 + x^2$$

Rules to Trig Substitutions

- Begin by replacing x with a trig function.

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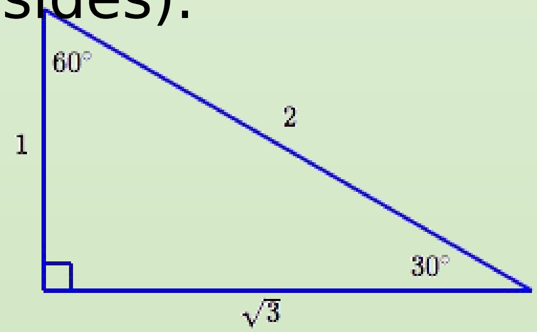
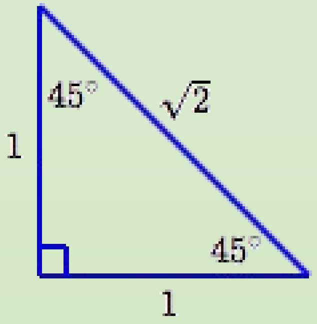
- Begin by replacing *x* with a trig function.
- Don't forget to also replace *dx* with the appropriate trig function.
- Use trig identities to solve the resulting integral.
- Be sure to rewrite your final answer in terms of *x*.
- *Know how to derive the corresponding right triangle in each of the three cases we consider below without memorizing them*

Review of Trigonometry

Special right triangles (ratio of sides):





Trig function inverse relationships diagram:

Rules to compute trig functions of right triangles:

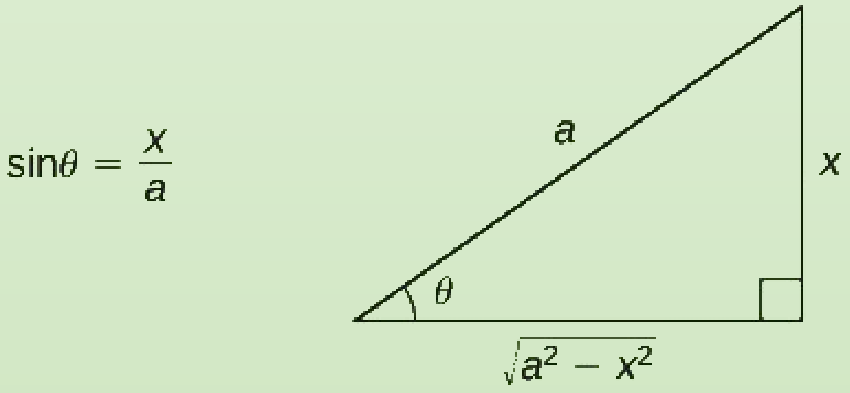
SOHCAHTOA

Sine **O**pposite **H**ypotenuse **C**osine **A**djacent **H**ypotenuse **T**angent **O**pposite **A**djacent

Credits for figures:
https://www.onemathematicalcat.org/Math/Precalculus_obj/trigValuesSpecialAngles.htm

Form 1: When the integral contains a term of the form $a^2 - x^2$,

use the substitution: $x = a\sin\theta$



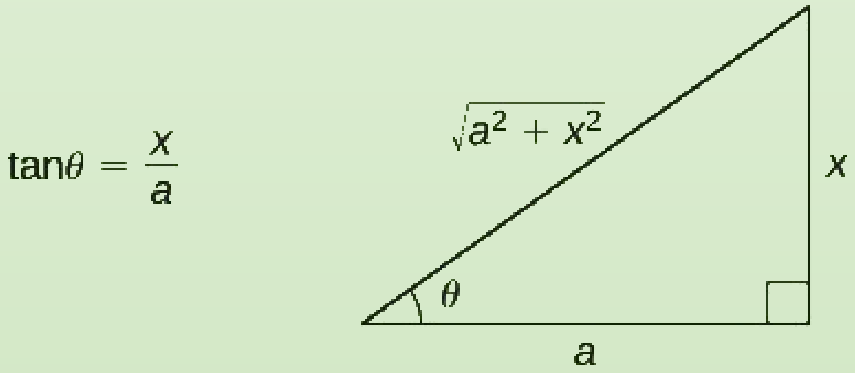
Credits for figure: https://math.libretexts.org/Bookshelves/Calculus (*Book:* OpenStax -> Techniques of Integration -> Trigonometric Substitution – Section 7.3)

Example 1:

Evaluate the integral: $\int \sqrt{4 - x^2}\, dx$

Form 2: When the integral contains a term of the form $a^2 + x^2$,

use the substitution: $x = a\tan\theta$



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Example 2:

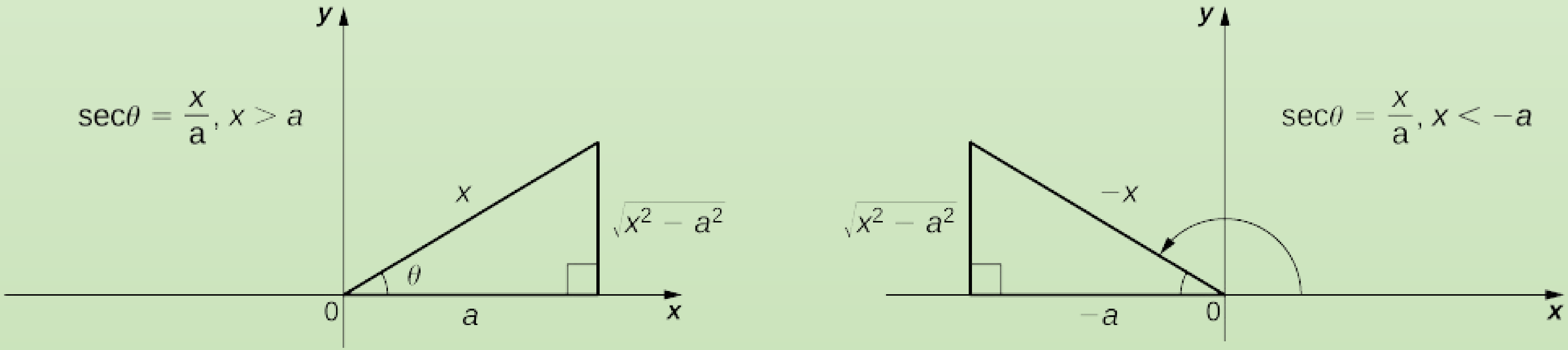
Evaluate the integral:

$$\int \frac{1}{\left(9+x^2\right)^{3/2}}\,dx$$

Form 3: When the integral contains a term of the form $x^2 - a^2$,

use the substitution:

$$x = a\sec\theta$$



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Example 3:

Evaluate the integral: $\int \frac{1}{x^4\sqrt{x^2 - 1}}\,dx$

Extra problem: Evaluate the integral: $$\int \frac{x}{\sqrt{x^2 - 3x + 7}}\,dx$$

Extra problem:

Evaluate the integral: $\int e^{4x}\sqrt{1+4e^{2x}}\,dx$